17. Calculate the spin-orbit splitting pattern for the ground state of a d^3 ion and a d^7 ion.

Equate the matrix elements of $\lambda \hat{L}\cdot\hat{S}$ and $\zeta_{nd}\sum_i \hat{l}_i\,\hat{s}_i$

	d^3	d^7
Ground state	^4F	^4F
M_L	3	3
M_S	$3/2$	$3/2$
Determinantal wave function $\Psi = \lvert\phi_1\,\phi_2\cdots\rvert$	$\lvert\overset{+}{2}\ \overset{+}{1}\ \overset{+}{0}\rvert$	$\lvert\overset{+}{2}\ \overset{-}{2}\ \overset{+}{1}\ \overset{-}{1}\ \overset{+}{0}\ \overset{+}{-1}\ \overset{+}{-2}\rvert$
Diagonal element $\langle\Psi\rvert\lambda\hat{L}\cdot\hat{S}\lvert\Psi\rangle = \lambda\langle\Psi\rvert\hat{L}_z\hat{S}_z\lvert\Psi\rangle = \lambda M_L M_S$	$\lambda\cdot 3\cdot\frac{3}{2}$	$\lambda\cdot 3\cdot\frac{3}{2}$
$\zeta_{nl}\langle\Psi\rvert\sum\hat{l}_i\hat{s}_i\lvert\Psi\rangle = \zeta_{nl}\sum_i\langle\phi_i\rvert\hat{l}_i\hat{s}_i\lvert\phi_i\rangle$ (Slater-Condon Rules) $= \zeta_{nl}\sum_i\langle\phi_i\rvert\hat{l}_{z_i}\hat{s}_{z_i}\lvert\phi_i\rangle$	$\zeta_{nd}\left[2\cdot\frac{1}{2}+1\cdot\frac{1}{2}+0\,\frac{1}{2}\right] = \frac{3}{2}\zeta_{nd}$	$\zeta_{nd}\left[2\,\frac{1}{2}+2\cdot\frac{1}{2}+1\cdot\frac{1}{2}+1\cdot\frac{-1}{2}+0\cdot\frac{1}{2}+(-1\cdot\frac{1}{2})+(-2\cdot\frac{1}{2})\right]$ $-\frac{3}{2}\zeta_{nd}$
	$\therefore\ \lambda = \frac{1}{3}\zeta_{nd}$	$\lambda = -\frac{1}{3}\zeta_{nd}$

Verify that for the ground state, in general $\lambda = \frac{1}{2S}\zeta_{nd}$ (less than half filled)

$\lambda = -\frac{1}{2S}\zeta_{nd}$ (more than " ")

Splitting pattern is, therefore,