

12. Show that $\langle \bar{\Phi}_n^a | \sum_j \sum_{i>j} \frac{1}{r_{ij}} | \bar{\Phi}_0 \rangle$

$$= \langle \bar{\Phi}_n^a | \sum_j (\hat{J}_j - \hat{K}_j) | \bar{\Phi}_0 \rangle$$

$$\bar{\Phi}_0 = | \phi_1 \phi_2 \dots \phi_n | \quad \text{where } \phi_1, \phi_2 \dots \text{ are spin-orbitals}$$

$$\text{L.H.S.} = \sum_j \sum_{i \neq j} \{ \langle \phi_i^{(1)} \phi_j^{(2)} | \frac{1}{r_{12}} | \phi_i^{(1)} \phi_j^{(2)} \rangle - \langle \phi_i^{(1)} \phi_j^{(2)} | \frac{1}{r_{12}} | \phi_j^{(1)} \phi_i^{(2)} \rangle \}$$

(Slater-Condon rules for 2-e operator for determinants differing by one spin-orbital)

$$= \sum_j \{ \langle \phi_i^{(1)} | \hat{J}_j^{(1)} | \phi_i^{(1)} \rangle - \langle \phi_i^{(1)} | \hat{K}_j^{(1)} | \phi_i^{(1)} \rangle \}$$

(Check using definition of \hat{J}_j and \hat{K}_j !)

$$= \langle \bar{\Phi}_n^a | \sum_j (\hat{J}_j - \hat{K}_j) | \bar{\Phi}_0 \rangle$$

(Slater-Condon rule for 1-e operator for determinants differing by one spin-orbital)

$$= \text{R.H.S.}$$