9. Let ψ_0, ψ_1, ψ_2 be the three lowest energy states of simple harmonic oscillator, $\hat{\mathcal{H}}_0$ being its Hamiltonian. In an electric field the Hamiltonian is $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + k\hat{x}$. Use the trial function $a_0\psi_0 + a_1\psi_1$ to calculate the ground state energy by variation method.

$q = \langle \psi_0 | \hat{x} | \psi_1 \rangle = ?$

$\hat{\mathcal{H}}$	$\lvert\psi_0\rangle$	$\lvert\psi_1\rangle$
$\langle\psi_0\rvert$	$\frac{1}{2}h\nu$	kq
$\langle\psi_1\rvert$	kq	$\frac{3}{2}h\nu$

The secular determinant upon expansion gives,

$$\left(\tfrac{1}{2}h\nu - E\right)\left(\tfrac{3}{2}h\nu - E\right) - k^2q^2 = 0$$

$$E^2 - 2h\nu E + \tfrac{3}{4}h^2\nu^2 - k^2q^2 = 0$$

Solve the quadratic equation to get for the ground state,

$$E = h\nu - \sqrt{k^2q^2 + \tfrac{1}{4}(h\nu)^2}$$

$$\sim \tfrac{1}{2}h\nu - \frac{k^2q^2}{h\nu}$$

How?