

(7) Find the probability of finding the electron farther away than the Bohr radius for the ground state of the hydrogen atom

$$R(r) = R_{1s}(r) = 2 e^{-r/a_0}$$

The ~~required~~ probability is beyond $t a_0$

$$4 \int_{t a_0}^{\infty} e^{-2r/a_0} r^2 dr$$

$$\int e^{-2r/a_0} r^2 dr = e^{-2r/a_0} \left(\frac{r^2}{-2} - \frac{2r}{4} + \frac{2}{-8} \right) \quad \left[\int x^2 e^{ax} dx = e^{ax} \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) \right]$$

$$\therefore 4 \int_{t a_0}^{\infty} e^{-2r/a_0} r^2 dr = (2t^2 + 2t + 1) e^{-2t}$$

The required probability is for $t = 1$

$$= 5 e^{-2} = 0.68$$

What is the electron density at the nucleus for the ground state?