6. Calculate $\langle r \rangle_{3d}$ for $O^{7+}$ ion

$$\langle r \rangle = \langle R(r)\,\Theta(\theta)\,\Phi(\phi) \mid \hat{r} \mid R(r)\,\Theta(\theta)\,\Phi(\phi) \rangle$$

$$= \int_0^\infty r^3\, R(r)^2\, dr$$

Show how!

Also Check that this is dimensionally correct.

$$R_{3d}(r) = \frac{4}{81\sqrt{30}}\, Z^{7/2}\, r^2\, e^{-Zr/3} \quad \text{in at. units}$$

$$\langle r \rangle = \frac{16}{81^2 \times 30}\, Z^7 \int_0^\infty r^7\, e^{-2Zr/3}\, dr$$

$$= \frac{16}{81^2 \times 30}\, Z^7 \times \frac{7!}{\left(\frac{2Z}{3}\right)^8} = \frac{16 \times 7! \times 3^8}{81^2 \times 30 \times 2^8 \times Z}$$

$$= \frac{21}{2Z} \text{ a.u.}$$

$$= \frac{21}{16} \text{ a.u} = 1.312 \text{ a.u} = 1.312 \times 0.5292$$

$$= 0.694\ \text{Å}$$

Calculate $r_{max}$ for this orbital.