3. Find the relation between $\langle \hat{T} \rangle$ and $\langle \hat{V} \rangle$ for the ground state of the harmonic oscillator

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$$\psi = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha x^2}{2}} \qquad \alpha = \frac{\mu \omega}{\hbar}, \quad \omega^2 = \frac{k}{\mu}$$

$$\langle \hat{V} \rangle = \frac{1}{2} k \int_{-\infty}^{\infty} x^2 \psi^2 \, dx$$

$$= \frac{1}{2} k \frac{\alpha^{1/2}}{\sqrt{\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{\alpha x^2}{2}} \, dx$$

$$\langle \hat{T} \rangle = \frac{\hbar^2}{2\mu} \int_{-\infty}^{\infty} \left(\frac{d\psi}{dx}\right)^2 dx \qquad \text{(see problem 1)}$$

$$\frac{d\psi}{dx} = \left(\frac{\alpha}{\pi}\right)^{1/4} \cdot - \frac{2\alpha x}{2} e^{-\frac{\alpha x^2}{2}}$$

$$\therefore \langle \hat{T} \rangle = \frac{\hbar^2}{2\mu} \left(\frac{\alpha}{\pi}\right)^{1/2} \alpha^2 \int_{-\infty}^{\infty} x^2 e^{-\frac{\alpha x^2}{2}} \, dx$$

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Show that the constant factors in the above two integrals are equal.

$$\therefore \langle \hat{T} \rangle = \langle \hat{V} \rangle$$