2. For a particle in a box calculate $\langle \hat{p} \rangle$ and $\langle \hat{T} \rangle$.

$$\psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l} \qquad \hat{p} = -i\hbar \frac{d}{dx}$$

$$\langle \hat{p} \rangle = -i\hbar \frac{2}{l} \int_0^l \sin \frac{n\pi x}{l} \frac{d}{dx} \sin \frac{n\pi x}{l} \, dx$$

$$= -i\hbar \frac{2}{l} \frac{n\pi}{l} \int_0^l \sin \frac{n\pi x}{l} \cos \frac{n\pi x}{l} \, dx$$

$$= 0$$

Could we have got this result without integrating? (Note that here we have an imaginary operator and a real wave function!)

$\langle T \rangle = T$, the eigen value, because ψ_n is an eigen function of $\frac{d^2}{dx^2}$. $\left(\hat{T} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right)$

$$T = \frac{n^2 \hbar^2 \pi^2}{2 m l^2}$$

The operator $\hat{p}$ commutes with $\hat{T}$. Does it mean that every eigen function of $\hat{T}$ will be an eigen function of $\hat{p}$?