16. Show that for a single particle moving in a potential $V(x)$, $\frac{\partial \langle p \rangle}{\partial t} = -\left\langle \frac{\partial V}{\partial x} \right\rangle$

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) \qquad \left[\hat{\mathcal{H}}, \frac{d}{dx}\right] = -\frac{dV}{dx}$$

Show how!

$$\frac{d\langle p \rangle}{dt} = -i\hbar \frac{d}{dt}\int \psi^* \frac{d}{dx}\psi \, dx$$

$$= \int -i\hbar \frac{d\psi^*}{dt}\frac{d\psi}{dx}\,dx + \int -i\hbar\, \psi^* \frac{d}{dt}\frac{d\psi}{dx}\,dx$$

$$= \int -i\hbar \frac{\partial\psi^*}{\partial t}\frac{\partial\psi}{\partial x}\,dx + \int -i\hbar\, \psi^* \frac{\partial}{\partial x}\frac{\partial\psi}{\partial t}\,dx$$

$$= \int \hat{\mathcal{H}}\psi^* \frac{\partial\psi}{\partial x}\,dx - \int \psi^* \frac{\partial}{\partial x}\hat{\mathcal{H}}\psi\,dx$$

Show how!

$$\left(\mathcal{H}\psi = i\hbar\frac{d\psi}{dt} \qquad \mathcal{H}\psi^* = -i\hbar\frac{d\psi^*}{dt}\right)$$

$$= \int \psi^* \hat{\mathcal{H}}\frac{d}{dx}\psi\,dx - \int \psi^* \frac{\partial}{\partial x}\hat{\mathcal{H}}\psi\,dx$$

$$= \int \psi^* \left(\hat{\mathcal{H}}\frac{d}{dx} - \frac{\partial}{\partial x}\hat{\mathcal{H}}\right)\psi\,dx$$

$$= -\int \psi^* \frac{dV}{dx}\psi\,dx = -\left\langle \frac{\partial V}{\partial x}\right\rangle$$

This shows that the expectation values obey Newton's law $\left(F = ma = \frac{dp}{dt} = -\frac{dV}{dx} \right)$