11. The polarizability of an atom, α, is related to the change in energy of the atom, ΔE, in an external electric field, F, such that $\Delta E = -\frac{1}{2} \alpha F^2$. Derive an equation for the ground state polarizability of an atom using perturbation theory.

Assuming an uniform electric field, F, along the x axis, the perturbation is $\hat{\mathcal{H}}' = -eF\hat{x}$

First order correction to the energy of the ground state $\psi_1^0 =$

$$-eF\langle \psi_1^0 | \hat{x} | \psi_1^0 \rangle$$

$$= 0 \quad \text{Why?}$$

Second order correction =

$$-e^2F^2 \sum_{i \neq 1} \frac{|\langle \psi_1^0 | \hat{x} | \psi_i^0 \rangle|^2}{E_i^0 - E_1^0}$$

(See problem 10)

∴ Correct to second order,

$$\alpha = 2e^2 \sum' |x_{1i}|^2 / (E_i^0 - E_1^0), \quad x_{1i} = \langle \psi_1^0 | \hat{x} | \psi_i^0 \rangle$$