1. Show that the expectation value of kinetic energy for a 1-dimensional system is

$$\langle T \rangle = \frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx .$$

$$\hat{T} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$$

$$\therefore \langle \hat{T} \rangle = -\frac{\hbar^2}{2m} \langle \psi | \frac{d^2}{dx^2} | \psi \rangle$$

$$= -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \psi^* \frac{d^2}{dx^2} \psi \, dx$$

$$\int_{-\infty}^{\infty} \psi^* \frac{d^2}{dx^2} \psi \, dx = \psi^* \frac{d\psi}{dx} \Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{d\psi}{dx} \frac{d\psi^*}{dx} dx$$

The first term on the R.H.S. is zero (why?)

$$\therefore \langle \hat{T} \rangle = \frac{\hbar^2}{2m} \int \left| \frac{d\psi}{dx} \right|^2 dx .$$

Generalise this to the x-component of T in three dimensions.