Tutorial Notes # 4. Bohr's Correspondence Principle

For very large values of the quantum number a quantum system will tend to behave like a classical system.

For example, ~~for~~ when n is large, the particle-in-a 1-d box wave function will have a large number of ~~closely~~ closely spaced maxima along the length of the box such that the probability density will tend to become equal everywhere — as for a classical particle. Since $E = \frac{n^2 h^2}{8 m l^2}$, for a macroscopic particle with macroscopic energy (m, l, E all large), n will be very large (because h is very small), so it will behave like a classical particle. No quantisation and ~~equal~~ uniform distribution within the box. Note that ~~for~~ if $\Delta E_n = E_n - E_{n-1}$, for large n,

$$\frac{\Delta E_n}{E_n} \sim \frac{1}{n}$$

Now consider the H atom wave function.

$$\hat{L}^2 \psi_{n,l,m} = l(l+1)\hbar^2 \psi_{n,l,m}$$

According to the correspondence principle for very large values of l, ~~the~~ $\hbar^2 l(l+1)$ should be identified with the total classical angular momentum of the electron in its orbit.

For $l = n-1$ and $m = l$ (highest allowed values for a given n), we have

$$\psi_{n,l,m} \approx \sim \rho^{n-1} e^{-\rho/n} \sin^{n-1}(\theta)\, e^{i(n-1)\phi}$$

The maximum in the radial part, ~~£~~ (at $\rho = n(n-1)$) as well as the maximum in the θ part (at $\theta = \pi/2$) will become more and more sharply defined as n increases. Therefore, for very large n the wave function tend to represent ~~a~~ the classical circular Bohr orbit.