Tutorial Notes # 1 Implicit and Explicit dependendence

★ If $V = V(x)$ and x depends on time, then V depends explicitly on x and implicitly on t.

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt}$$: V does not change with time unless x also has changed!

If V depends explicitly on x and t, then

$$V = V(x,t) \implies \frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial t}$$

$\therefore$ If $V = V(x)$, then $\frac{\partial V}{\partial t} = 0$, but $\frac{dV}{dt}$ may be nonzero

Similarly, for example, if $x = t^2$, then

$$\dot{x} = 2t \qquad \frac{d\dot{x}}{dx} = \frac{1}{t}, \text{ but } \frac{\partial \dot{x}}{\partial x} = 0$$

★ When is this true?

$$\frac{d^2f}{d\theta^2} = \frac{d^2f}{dx^2}\left(\frac{dx}{d\theta}\right)^2 \rightarrow \text{only when } x = \text{const.}\ \theta$$

In general $$\frac{d^2f}{d\theta^2} = \frac{d^2f}{dx^2}\left(\frac{dx}{d\theta}\right)^2 + \frac{df}{dx}\frac{dx}{d\theta}\frac{d}{dx}\left(\frac{dx}{d\theta}\right)$$

Note that $\frac{d}{dx}\frac{dx}{d\theta} \neq \frac{\partial^2 x}{\partial x \partial \theta} = 0$! (x and θ are not independent variables)

* During partial differentiation, it is important to keep track of what is kept fixed!

For example, $r^2 = x^2 + y^2 + z^2$

$$\therefore \; 2r\left(\frac{\partial r}{\partial x}\right)_{y,z} = 2x \qquad \therefore \left(\frac{\partial r}{\partial x}\right)_{y,z} = \frac{x}{r} = \sin\theta\cos\phi$$

But, $x = r\sin\theta\cos\phi$

$$\left(\frac{\partial x}{\partial r}\right)_{\theta,\phi} = \sin\theta\cos\phi$$

Note that $\left(\frac{\partial r}{\partial x}\right)_{y,z} \neq \dfrac{1}{\left(\frac{\partial x}{\partial r}\right)_{\theta,\phi}}$